SOME TOPICS ON F-THRESHOLDS

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(This is a joint work with C. Huneke and S. Takagi.)

The notion of F-threshold $c^J(a)$ is introduced by Mustaţă for pairs of ideals in a Noetherian ring of characteristic $p > 0$ with $a \subset \sqrt{J}$.

Our main concern with this notion is the following conjecture.

**Conjecture 0.1.** Let $(A, \mathfrak{m})$ be a Noetherian local ring of characteristic $p > 0$ of dimension $d$, $J$ be a parameter ideal of $A$ and $a$ be a $\mathfrak{a}$ primary ideal. Then

$$e(a) \geq \left( \frac{d}{c^J(a)} \right)^d e(J)?$$

where $e(J)$ (resp. $e(a)$) denotes the multiplicity of $J$ (resp. $a$).

This conjecture is true if $A = \oplus_{n \geq 0} A_n$ is a graded ring over Artinian local ring $A_0$ and both $J$ and $a$ are generated by full system of homogeneous parameters.

Also, we discuss when the equality holds in our conjecture.

Recently, the relation of F-threshold with the F-jumping number is found.

**Definition 0.2.** For every ideal $J \subseteq A$ such that $a \subseteq \sqrt{J}$, the $F$-jumping number $\text{fjn}^J(a)$ of $a$ with respect to $J$ is defined to be

$$\text{fjn}^J(a) = \inf\{t \geq 0 \mid \tau(a^t) \subseteq J\}.$$ 

Where $\tau(a^t)$ is the generalization of test ideal defined by N.Hara and K.-i. Yoshida. This notion is known to have strong connection with multiplier ideals in algebraic varieties over a field of characteristic 0. Then our F-threshold has the following characterization.

**Theorem 0.3.** Suppose that $A$ is an equidimensional local ring of characteristic $p > 0$ and $J$ is an ideal generated by a full system of parameters for $A$. Assume in addition that $A$ is Gorenstein and $A_P$ is $F$-rational for all prime ideals $P$ not containing $a$. Then

$$\text{fjn}^J(a) = c^J(a).$$

In terms of this characterization, our conjecture on multiplicity and F-thresholds is equivalent to the following conjecture on core of ideals.

**Conjecture 0.4.** Let $(A, \mathfrak{m})$ be a 2-dimensional $F$-rational Gorenstein ring, let $J$ be a parameter ideal in $A$ and $a$ be an integrally closed $\mathfrak{m}$ primary ideal. If $J \supset core(a)$, then $e(a) \geq e(J)$?
References


